

# Correlation of Modified Transient Plane Source Technique and Modified Hot Wire Results with Traditional Techniques for Thermal Conductivity Measurement

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## Abstract

The Modified Transient Plane Source (MTPS) technique developed by C-Therm Technologies Ltd. provides a rapid convenient test method for the thermal conductivity characterization of solid, liquid, powder and paste materials under 100 W/mK and is designed to provide simple, highly accurate thermal characterization for lab, quality control and production environments. This paper compares MTPS testing results with other traditional techniques for thermal conductivity measurement and the modified hot-wire (MHW) technique recently developed at the University of New Brunswick for testing low-conductivity solid materials. A statistical comparison of results is presented on various different solid and liquid materials in order to establish the equivalency between the thermal conductivity measurement techniques. Materials tested include: LAF foam, PYREX glass, PDMS (poly dimethyl siloxane silicone) and water gel.

The MTPS measurements were performed using the TCi Thermal Conductivity Analyzer commercially produced by C-Therm Technologies. Five to six separate tests were done with two groups of three tests done with 1 minute cooling times between each.

The modified hot wire method testing was performed at the University of New Brunswick (UNB) on a prototype apparatus built based on a modification of the line source technique with the hot wire probe immersed in the test materials with measurement times of one minute followed by cooling times of five minutes. Three separate runs were made per material. A linear regression result of  $y = 1.0981 x + 0.0227$ ;  $R^2 = 0.9977$  was obtained with y being the MTPS reading and x the MHW reading.

## Introduction

The main focus of this work is to demonstrate a correlation between the MTPS technique developed by C-Therm Technologies, a variation of the hot wire technique developed at the University of New

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Brunswick and other traditional techniques. Statistical comparisons of the thermal conductivity measuring techniques are to be provided.

The MTPS method is based on a continuous plane heat source (Carslaw and Jaeger pp 262-3) with further developments as one can see in the reference: C-Therm (see C-Therm reference cited). The hot wire method, utilizes a continuous line heat source solution (Carslaw and Jaeger pp 261-2). Here the hot wire is either placed between two pieces of solid material, or immersed in the case of a liquid sample.

The samples used in this correlation study are LAF foam, PYREX glass, PDMS and water gel (or gelatin).

### **Test Methods**

**MODIFIED HOT WIRE:** In the UNB lab the hot wire probe is immersed in the testing material. For the case of solid samples the wire is sandwiched between two identical pieces of the sample which are then clamped together by a 250 gram weight to ensure good contact. For the liquid and gel measurements, the probe is immersed by simply placing it in a test tube that is filled with the material.

An electrical circuit supplies a constant heating rate (typically 1 Watt per meter of wire length) to the wire and also reads the wire resistance as time progresses. The RCT equation is used to convert wire resistance to wire temperature (Boylestad et. al.):

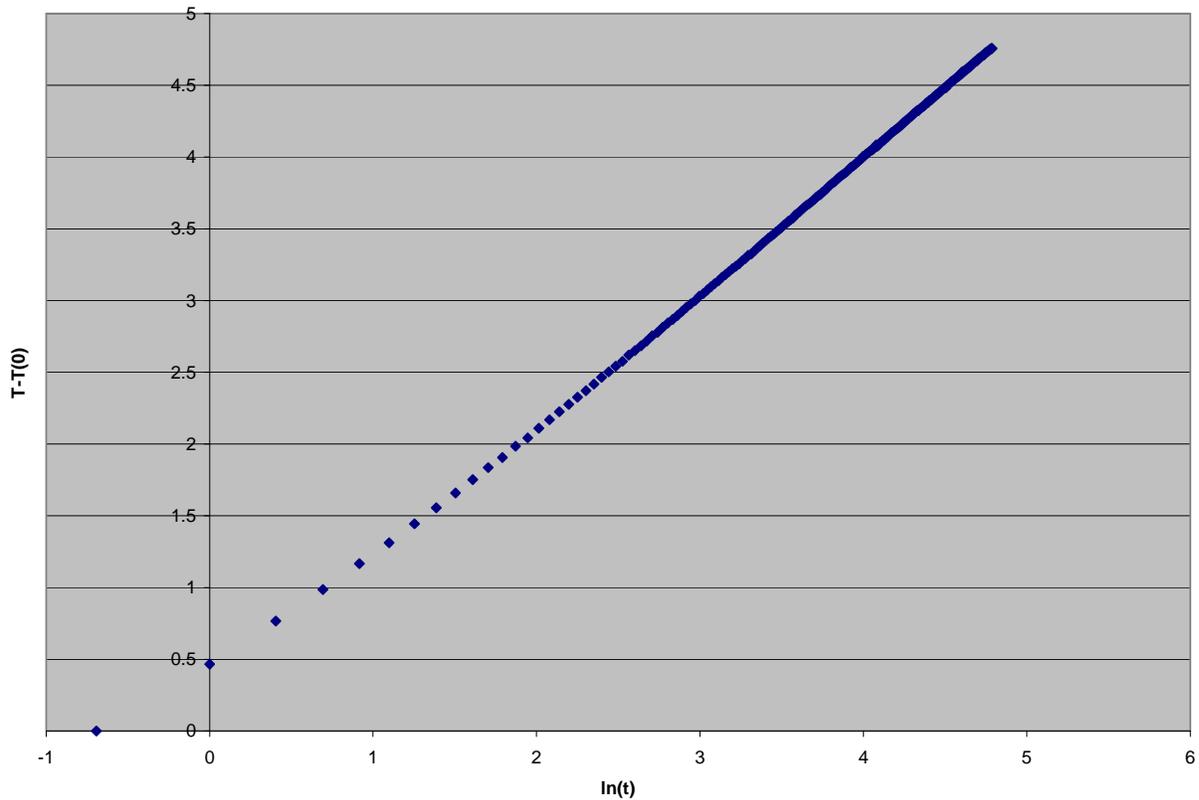
$$T^{\circ}\text{C} = (R_T / R_0 - 1) / \alpha \quad (1)$$

Here:  $R_T$  is the measured probe resistance at a given time,  $R_0$  is the wire resistance at zero Celsius, and  $\alpha$  is the wire temperature coefficient of resistance obtained by calibration in a high resolution controlled temperature water bath. The resulting temperature is obtained in Celsius degrees.

This temperature rise is then plotted against the natural logarithm of time yielding a straight line of constant slope for a uniform sample material (Carslaw and Jaeger). The thermal conductivity is given as:

$$k_{\text{meas.}} = Q' / (4 * \pi * S) \quad (2)$$

Here:  $k_{\text{meas.}}$  is the measured thermal conductivity in W/mK,  $Q'$  is the line source heat input rate and  $S$  is the slope of temperature vs. natural log of time taken in the linear zone typically between  $\ln(t)=2$  and  $\ln(t)=3$ .



**Figure 1: Temperature Rise Curve for LAF Foam Thermal Conductivity Measurement**

MTPS TEST METHOD: The TCi sensor has a central heater/sensor element in the shape of a spiral surrounded by a guard ring. The guard ring generates heat in addition to the spiral heater, thus, approximating a one-dimensional heat flow from the sensor into the material under test in contact with the sensor. This simply makes this a plane source of heat whereby the solution is given in Carslaw and Jaeger. The voltage drop on the spiral heater is measured before and during the transient. The voltage data is then translated into the effusivity value of the tested material. Detailed developments from the plane source solution as well as the relevant working equation are provided (C-Therm):

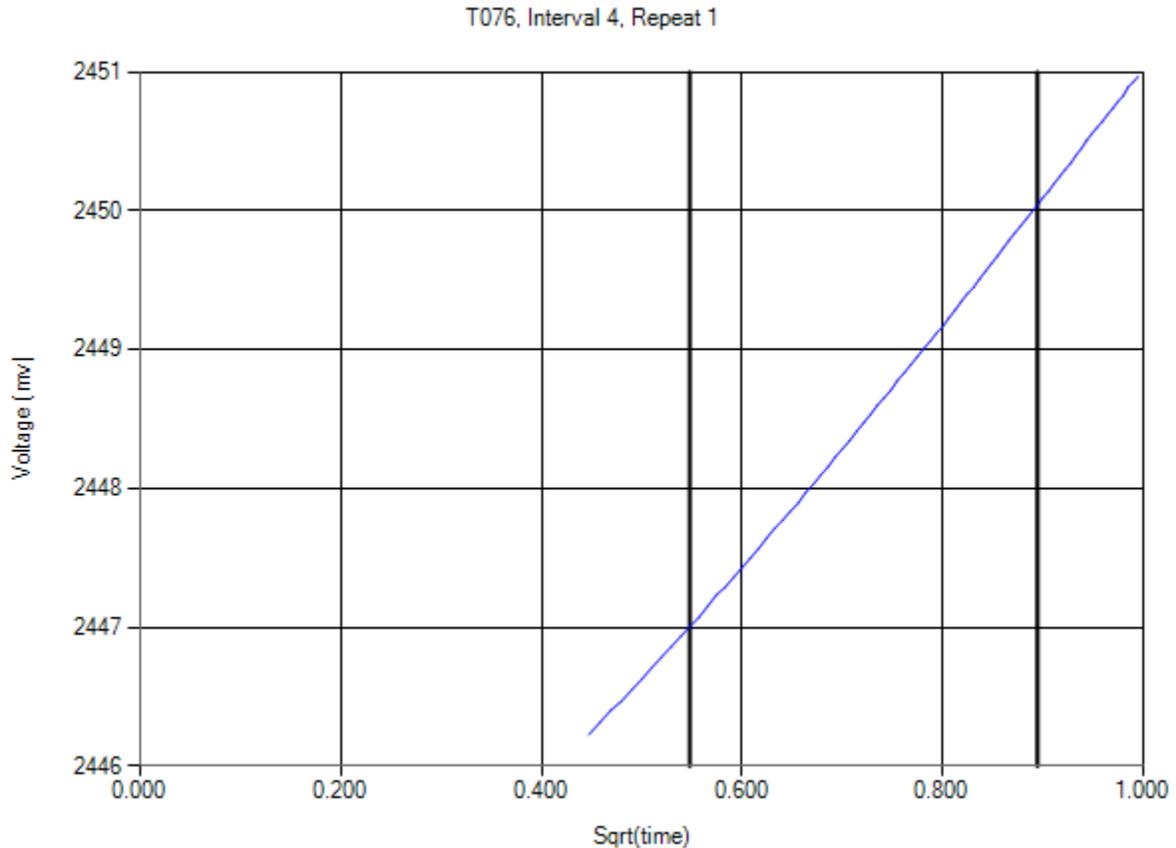
$$E_{\text{sample}} = 1.1284 * Q'' / S - E_{\text{sensor}} \quad (3)$$

Here  $E_{\text{sample}} = (k * \rho * Cp)_{\text{sample}}^{1/2}$  is the thermal effusivity of the sample in  $Ws^{1/2}/m^2K$ ;  $E_{\text{sensor}} = (k * \rho * Cp)_{\text{sensor}}^{1/2}$  is the thermal effusivity of the sensor material,  $Q''$  is the heat input of the per unit area of the probe and  $S$  is the slope of the temperature rise vs. the square root of time or  $S = dT/d(t^{1/2})$ .

The temperature rise is obtained in the same manner as was done for the hot wire utilizing the RCT equation.

The conductivity is calculated from the voltage data by C-Therm's patented iterative method ( $m^*$ ).

A sample of the voltage drop curve is shown in Figure 2:



**Figure 2: Voltage Drop Curve for MTPS Probe Thermal Conductivity Measurement for PYREX**

The system automatically compensates for variations in sensor temperature, thus enabling reliable measurements at a wide range of temperatures (-50°C to +200°C).

For additional information please refer to Patent # 6,676,287: Direct thermal conductivity measurement technique, Patent ## 20040165645: Method and apparatus for monitoring substances and Patent # 5,795,064: Method for determining thermal properties of a sample.

Among the differences between the MHW and MTPS technique:

The MTPS sensor is supported by an insulative backing material and offers a one-sided interfacial test interface vs. a two-sided interfacial contact (or immersing the probe in fluid) the MHW employs in sandwiching the sample around the probe

The MTPS (TCi Thermal Conductivity Analyzer) is factory-calibrated and provides direct thermal conductivity data output on the samples based on the iterative  $m^*$  algorithm

## Results and Discussion

Table 1 outlines the resulting thermal conductivity measurements, showing the correlation between the two measuring techniques:

**Table 1: Thermal Conductivity Measurement Results in W/mK**

Material	k (MHW)	k (MTPS)	k (lit.)	% diff (MHW)	% diff (MTPS)	% disagreement
PYREX****	1.08	1.17	1.135	5.36	3.04	8.39
PDMS**	0.189	0.162	0.159	17.24	1.87	15.38
LAF***	0.0848	0.0880	0.086	1.41	2.30	3.70
Watergel*,**	0.621	0.626	0.627	0.96	0.16	0.80

\*Modified Hot Wire, value from Picot Laboratory Notes, 2002

\*\*Hot Wire, Bashar 2005

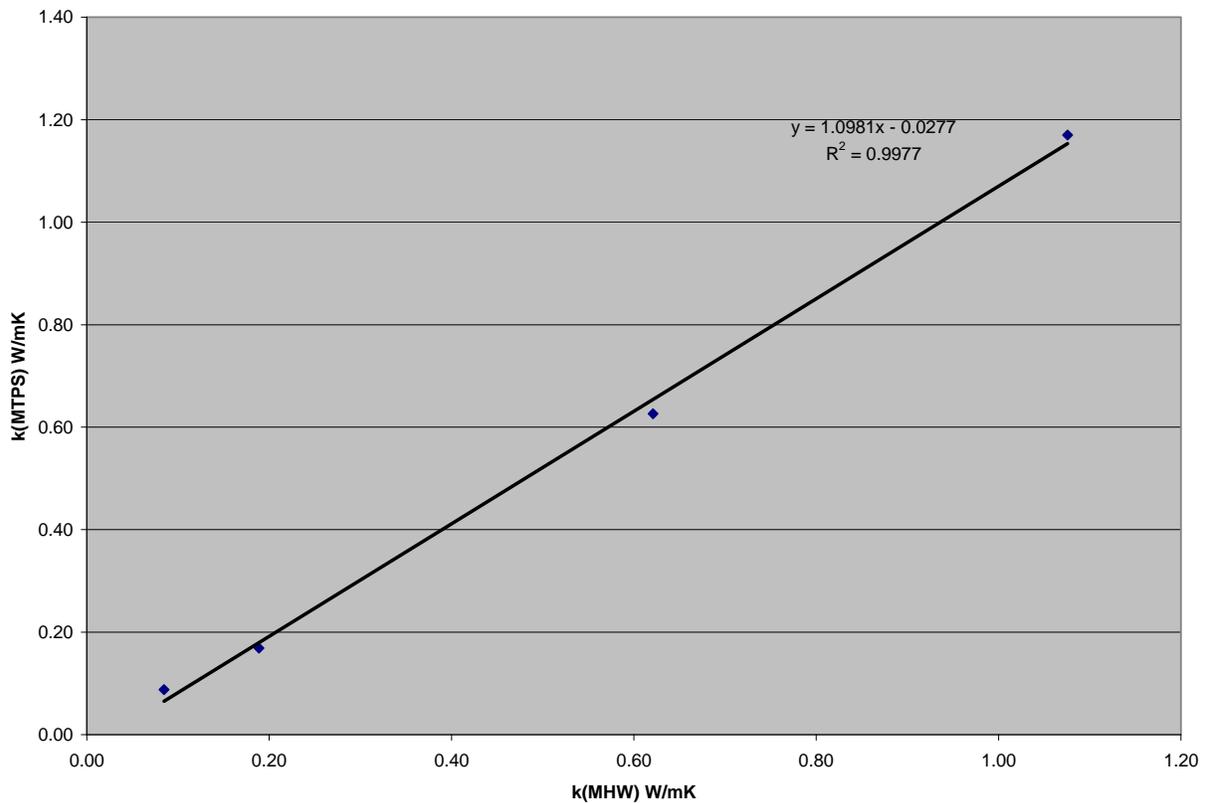
\*\*\*Transient Plane Source Method, 3<sup>rd</sup>-Party Lab Testing Report

\*\*\*\*Guarded Hot Plate, BCR Information, Certification Report for a Pyrex Reference Sample

The modified hot wire results are an average of three separate experiments. The MTPS results span several experiments (typically 4 to 5 minutes). Both methods showed sufficient precision.

The MTPS results vs expected literature values obtained by more traditional methods are consistent with the stated accuracy specification for the TCi thermal conductivity analyzer (<5%).

Regression analysis of k(MTPS) as y versus k(MHW) is shown in figure 3:



**Figure 3: Correlation of the Two Thermal Conductivity Methods**

From here the linear regression is obtained to be:

$$y = 1.0981 x - 0.0227 \quad ; \quad R^2 = 0.9977 \quad (4)$$

There is an 8.39% disagreement for the case of Pyrex and an 15.39% disagreement for the PDMS; the other two materials show an average disagreement of 2.25%.

The MHW data have been adjusted for end loss effects which were less than 1%. The watergel values of 0.627 and 0.621 are the result of this, the first being the uncorrected result of Bashar, done in the UNB lab earlier. The PDMS results are apparently affected by natural convection caused by the temperature rise of the measuring element. The error is more prominent in the case of the vertically oriented HW than the inverted horizontal MTPS, which is consistent with convection theory. The MTPS also employs a shorter measurement time of 0.8 second limiting the impact of convection.

Table 1 does prove that the two different methods are capable of producing very comparable results; the two probes correlate quite well within an average error of 5% difference between the two methods.

### **Concluding Remarks**

This study shows a successful correlation between two different probes developed in two different organizations. The immersed modified hot wire probe, developed at the University of New Brunswick and the Modified Transient Plane Source probe developed by C-Therm technologies show analogous readings for the four different samples used.

### **References Cited**

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